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INDEXING, COINTEGRATION AND EQUITY MARKET REGIMES

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ABSTRACT

This paper examines, from a market efficiency perspective, the performance of a simple dynamic equity indexing strategy based on cointegration. A consistent 'abnormal' return in excess of the benchmark is demonstrated over different time horizons and in different real world and simulated stock markets. A measure of stock price dispersion is shown to be a leading indicator for the abnormal return and their relationship is modelled as a Markov switching process of two market regimes. We find that the entire abnormal return is associated with the high volatility regime as the indexing model implicitly adopts a strategic position that pays off during market crashes, whilst effectively tracking the benchmark in normal market circumstances. Therefore we find no evidence of market inefficiency. Nevertheless our results have implications for equity fund managers: we show how, without any stock selection, solely through a smart optimization that has an implicit element of market timing, the benchmark performance can be significantly enhanced. Copyright © 2005 John Wiley & Sons, Ltd.

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KEY WORDS: Cointegration; dispersion; efficient market hypothesis; equity markets; index tracking; Markov switching; market timing

1. INTRODUCTION

The phenomenon of equity indexing has attracted considerable interest in the last 10 years, from both academics and practitioners. The passive investment industry as a whole has witnessed a remarkable growth with a huge number of funds pegging their holdings to broad market indexes such as SP500. The very reason for adopting a passive strategy rests in a belief in market efficiency, which provides the theoretical foundation of indexing. However, one needs to make the distinction between a pure index fund, managed to replicate the performance of the market portfolio or benchmark exactly, and strategies such as enhanced index tracking, that extend it into active management. The latter are constructing well-diversified portfolios that have a stable relationship with the benchmark and try to take advantage of some pockets of market inefficiency.

According to Jensen's (1978) definition of efficient markets, a trading strategy producing consistent risk-adjusted economic gains, after properly defined transaction costs and over a sufficiently long period of time, is evidence against the efficient market hypothesis (EMH). This approach to market efficiency, as compared to earlier ones, has the advantage of testability and has subsequently generated a great deal of empirical research. Most of these studies, employing for example technical analysis and filter rules (Alexander, 1964; Fama and Blume, 1966), have shown that even if different trading strategies are successful before transaction costs, after accounting for such costs the profits vanish. Published evidence of trading profitability, after properly defined transaction costs, is rather scarce, Lakonishok and Vermaelen's (1990) paper being one of the very few to document the profitability of some trading rules designed to exploit anomalous price behaviour.

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In this paper, we investigate the performance of a very simple indexing strategy that has recently come to the attention of many fund managers—the cointegration based index tracking, introduced by Alexander (1999) and further developed as a statistical arbitrage by Alexander and Dimitriu (2005). In several real world and simulated stock market universes we find that, out of sample and after transaction costs, a portfolio constructed on a cointegration relationship with a price weighted index produces positive return in excess of its benchmark, which we call the 'abnormal return'. To note, the terminology employed is not meant to suggest, *a priori*, market inefficiency. The aim of this paper is to investigate the statistical properties of the abnormal return and the extent to which this can be considered evidence against the EMH.

We find that the pattern of the abnormal return exhibits a pronounced time-variability: periods of stationary, zero mean returns alternate with periods during which positive returns are consistently accumulated. Since the only information used to construct the portfolio is a history of stock prices, the cause of the abnormal return should be linked to the allocation weights and the time-variable features of stock prices. We introduce a new measure of stock price 'cohesion', which we call *index dispersion*, and find that this is a leading indicator of the abnormal return.

Throughout the analysis we justify the conclusions drawn from real world and simulated stock and index prices. In the real world universe of the Dow Jones Industrial Average (DJIA), we document a significant non-linear relationship between the abnormal return and the lagged dispersion, which, however, shows considerable time variability in parameters. To address this issue we estimate a Markov switching model for the abnormal return and find strong evidence of a latent state variable, which determines the form of the linear relationship between the abnormal return and the stock prices dispersion. The Markov switching model indicates the presence of two regimes having very different characteristics. The entire abnormal return is shown to be associated with the high volatility regime, and in this context we cannot rule out the presence of a temporary, hidden risk factor in the indexing strategy, such as an implicit market timing bet that prices will revert to historical equilibrium levels in due course.

To summarize our contributions, this paper re-examines market efficiency from a new perspective, focusing on potential stock price anomalies identified through cointegration and on the mechanism generating the abnormal returns. Our findings have wide implications for the passive investment industry. We show that, without any stock selection or *explicit* timing attempts, which are attributes of active management, solely through a simple optimization on prices, the benchmark performance can be significantly enhanced, even after accounting for transaction costs. Moreover, the strategy can be applied to replicate any type of value or capitalization weighted benchmark, not only wide market indexes.

In the remainder of the paper, Section 2 reviews the cointegration based tracking strategy and demonstrates the abnormal return in real world and simulated stock universes. Section 3 examines the factors that might contribute to this abnormal return, identifying an implicit market timing element of the strategy where abnormal returns will be made if the market crashes after a speculative bubble. Section 4 demonstrates a strong relationship between the abnormal return and the lagged dispersion in the DJIA universe of stocks. It is clear, however, that this relationship is not stable over time and the evidence of a structural break in October 2000 is conclusive. Section 5 shows that the simple indexing model can, in certain circumstances, develop a strong element of strategic positioning. That is, there is an implicit 'market timing' element in the strategy. We show how this explains the reversal in sign of the relationship between dispersion and the abnormal return after the structural break. In Section 6 we estimate a Markov switching model of dispersion. The intuition behind our results, and the reasons why virtually all abnormal returns are gained in the high volatility regime, are explored in Section 7. Here we also show that a strong association between the credit spread and the probability of the high volatility regime adds further support to the strategic positioning inherent in a cointegration based tracking strategy. Section 8 makes statistical inferences and examines the predictive power of the Markov switching model. Although we can demonstrate substantial gains from actively trading on the model, these gains are unlikely to outweigh the significant costs. Section 9 discusses the implications of our finding for market efficiency. Finally, Section 10 summarizes and draws the main conclusions.

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2. THE COINTEGRATION TRACKING PORTFOLIO

The focus of this paper is the performance of a very general form of an indexing model based on cointegration (Alexander, 1999), which allows the replication of all types of benchmarks, with different numbers of stocks. The rationale for constructing portfolios based on a cointegration relationship with the benchmark, rather than correlation, rests on the following features of cointegration: the price difference between the benchmark and the replica portfolio is, by construction, stationary; the stock weights, being based on a large amount of history, have an enhanced stability; finally, there is a full use of the information contained in level variables such as stock prices. Moreover, a cointegration relationship between a benchmark and a portfolio comprising all or only part of its stocks is always easy to find when benchmarks are equally weighted, price weighted or capitalization weighted, because the benchmark is just a linear combination of stock prices.

The basic cointegration model for a tracking portfolio containing all the stocks included in the benchmark at a given moment is a regression of the form:

$$\ln(B_t) = c_1 + \sum_{k=1}^{n} c_{k+1} \ln(P_{k,t}) + \varepsilon_t \tag{1}$$

where B denotes the price of the benchmark, which is reconstructed historically based on its current membership and weights, P_k is the price of stock k and n is the total number of stocks included in the benchmark.[1] All variables in the model, apart from the error term, are integrated of order one.[2] The specification of the model in natural log variables has the advantage that, when taking the first difference, the expected returns on the portfolio will equal the expected returns on the benchmark, provided that the tracking error is a stationary process.

We note that the application of ordinary least squares (OLS) to non-stationary variables as in (1) is only valid in the special case of a cointegration relationship, that is if and only if the residuals in (1) are stationary. Testing for cointegration is, therefore, essential for constructing cointegration optimal tracking portfolios. The Engle–Granger (1987) methodology for cointegration testing is particularly appealing in this respect for its intuitive and straightforward implementation. Moreover, its well-known limitations (small sample problems, asymmetry in treating the variables, at most one cointegration vector) are not effective in our case. The estimation sample is typically set to at least three years of daily data, there is a strong economic background to treat the benchmark as the dependent variable, and identifying only one cointegration vector is sufficient for our purposes. Further to estimation, the OLS coefficients in model (1) are normalized to sum up to one, thus providing the composition of the tracking portfolio.

The starting point of this paper is the observation that, in the DJIA stock universe, the simple cointegration indexing model (1) fails to track the benchmark in particular down market circumstances, where it generates significant abnormal returns (Alexander and Dimitriu, 2005). To add more empirical support to this finding, we have conducted simulations on random subsets of stocks in the FTSE100, CAC40 and SP100 universes using daily data from 4th January 1997 to 31st December 2001. In each universe, we have randomly drawn 100 portfolios comprising a fixed number of stocks (50 for FTSE, 25 for CAC and 80 for SP100) and determined a price weighted benchmark for each portfolio. Each of the 300 benchmarks was tracked with a cointegration optimal portfolio comprising all the stocks included in that particular benchmark. The optimal weights were rebalanced every 10 trading days based on the new OLS coefficients of the cointegration regression (1), re-estimated over a fixed-length rolling calibration period of three years of daily data preceding the portfolio construction moment. In between rebalancing, the portfolios were left unmanaged (i.e. the number of stocks is kept constant) and evaluated, based on the daily closing prices of the stocks.

Table 1 reports the average annual excess return over the index over the period 1997 to 2001 for each stock universe, and for comparison, the results obtained in the DJIA universe by Alexander and Dimitriu (2005). The total excess return over the four-year period is positive in all universes, but clearly not uniformly distributed in time. For instance, in the case of the FTSE simulated indexes the largest excess return is in 2000, at almost 5%, while in the case of SP100 simulated indexes, the largest excess return

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Table 1. Annual returns

(A) Excess returns to cointegration tracking						
	1997	1998	1999	2000	2001	Overall
DJIA	1.71%	−1.00%	1.46%	2.08%	5.61%	7.82%
FTSE	−0.18%	−0.64%	0.92%	4.77%	0.34%	5.21%
CAC	1.19%	1.16%	−0.04%	1.22%	−0.13%	3.41%
SP100	0.73%	−1.68%	−1.84%	1.00%	4.59%	2.79%
(B) Benchmark returns						
	1997	1998	1999	2000	2001	
DJIA	20.41%	14.93%	22.49%	−6.37%	−7.37%	
FTSE	19.86%	8.24%	8.35%	7.32%	−11.71%	
CAC	8.21%	27.26%	38.44%	0.45%	−21.79%	
SP100	26.12%	19.04%	18.32%	1.34%	−13.13%	

occurs during 2001, amounting to 4.6%. Moreover, comparison with the benchmark returns in Table 1B indicates that the cointegration tracking strategies are making higher returns in down markets. These results do provide evidence of abnormal returns from the cointegration tracking strategy, with a consistent time-variability pattern across different markets. However, without an in-depth understanding of the mechanism driving it, one cannot exclude the hypothesis that this abnormal return is sample-dependent, despite the fact that it is identified in different stock universes.

3. ABNORMAL RETURNS, INDEX DISPERSION AND SPECULATIVE BUBBLES

For reasons of space we now restrict the analysis to the DJIA universe, using daily close prices for the 30 stocks in the DJIA as of December 2001 and a longer sample, starting January 1990. We estimate the out of sample performance of a portfolio constructed based on model (1) with a rolling three-year calibration period and 10-day recalibration/rebalancing frequency. The cumulative daily abnormal return during the entire sample period is shown in Figure 1 and amounts to 11.6% before transaction costs.[3]

In order to understand the sources of the over-performance we perform a standard Fama and French (1993) analysis at a monthly frequency, using size, value and momentum factors. We find no evidence of a size or value component in our strategy. The momentum factor has a significant relationship with the tracking portfolio's abnormal return, the two being negatively correlated. Such a relationship is to be expected, as any price weighted benchmark has an implicit momentum component, i.e. it overweights stocks that have recently increased in price and underweights stocks that have recently declined in price, relative to a portfolio based on historical averages such as the cointegration based tracking portfolio. However, the negative momentum factor premium explains only 30% of the abnormal return, even at a monthly frequency.

A very noticeable feature in Figure 1 is the time variability of the excess return, which is far from being uniformly accumulated throughout the data sample. Periods of stationary excess returns alternate with periods during which abnormal returns are consistently accumulated. Moreover, a visual inspection indicates that the periods during which most of the abnormal return is accumulated coincide with the main market crises during the sample period: the Asian crisis, the Russian crisis and the technology market crash. However, at a daily frequency, the correlation between the excess returns and changes in several measures of volatility was not found to be statistically significant. Therefore, a positive co-dependency between abnormal returns and a risk factor remains likely, but the risk factor is either not proxied by the benchmark volatility, or/and the co-dependency is more general than a simple, linear correlation.

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Figure 1. Cumulative abnormal return.

Conceptually, the over-performance of the cointegration portfolio has to be connected to the portfolio weighting system and its relationship with stock price dynamics. There are two sources of difference between the benchmark weights and the tracking portfolio weights: (a) tracking portfolio weights are constructed on natural log prices instead of prices, we call this the *log effect*; and (b) they reflect historical equilibrium relationships corresponding to the sample period used to calibrate the portfolio, rather than just the present relationship implied by today's prices, as in the price weighted benchmark. We call this the *historical prices* effect.

3.1. *Log prices effect*

The use of log rather than raw prices in constructing the tracking portfolio weights produces deviations from the benchmark weights which are proportional to the dispersion of raw prices. This is simply the effect of applying a non-linear rescaling: if the raw prices were in the same close range then applying a log transformation would have a similar impact on all of them. However, if there is a significant dispersion in raw prices the natural logarithm will no longer produce a uniform transformation. Therefore a significant difference between the benchmark weights and the portfolio weights can occur when the dispersion of stock prices increases.

This clear-cut result motivates our study of *index dispersion*, i.e. the cross-sectional standard deviation of the prices across their mean (which is the price weighted benchmark), defined as:

$$d_t = \sqrt{\sum_{k=1}^{n} ((P_{k,t} - B_t)/B_t)^2/n} \tag{2}$$

For computing the time series of index dispersion all stock prices are rescaled to be equal to 100 at the beginning of the period, the dispersion series therefore starting from zero. In Figure 2 we plot the time series of dispersion in the DJIA. After a steady increase the dispersion increased substantially at the beginning of the technology sector boom, due to the sharp increase in the price of technology stocks and a relative decline in price of other sectors. The highest dispersion occurred at the beginning of 2000, but since then the dispersion has decreased, most obviously during the crash of the technology bubble. We have found that index dispersion in most major equity markets (either capitalization or equally weighted) had followed a similar pattern over this period.

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Figure 2. Price dispersion in DJIA.

3.2. *Historical prices effect*

The second source of differences between the tracking portfolio and benchmark weights is that an historical sample is used to calibrate the cointegration portfolio. The price weighted benchmark reflects only the current ranking of stocks while the cointegration portfolio is based on an 'historical equilibrium', i.e. a price equilibrium estimated from a long historical sample. As already pointed out, this explains why the momentum factor has a lower loading for the tracking portfolio compared with the benchmark. The historical prices effect can be interpreted as an implicit 'market timing' bet of the cointegration tracking portfolio. That is, when the current price of a stock deviates from its historical equilibrium price the tracking portfolio 'bets' that the price will revert back to the equilibrium in the future. In ordinary market circumstances the cointegration portfolio will track the benchmark accurately. However, during a bubble formation, for example, the normal equilibrium mechanism breaks down as the prices of some stocks rise far above the price implied by the historical equilibrium relationship. The tracking portfolio will underweight such stocks during the formation of the bubble, implicitly betting on a reversal during the bubble burst.

We have tested this hypothesis by generating many simple two-stock scenarios in which one stock is experiencing a periodically collapsing speculative bubble. The bubble formation and collapse is modelled as in Blanchard and Watson (1982). A common feature of all such simulations is the clear-cut performance of the tracking portfolio during the bubble crash period. Figure 3 shows fours sets of simulated prices, together with the cumulative abnormal return generated by the tracking portfolio.[4] Most simulated portfolios underperform the price weighted benchmark during the bubble formation period, to some extent. But note that when there are many stocks in the universe there may be no under-performance at all, for the reasons given in Section 5 below. All simulated portfolios significantly outperform the benchmark at the time of the crash. This is because they have become substantially underweighted on the bubble stock, relative to the benchmark. This confirms our hypothesis that the use of historical prices in calibrating the tracking portfolio represents an implicit market timing bet, and that there is a clear connection between the performance of the cointegration based tracking and the presence of phenomena such as speculative bubbles in equity markets.

4. A BASIC TIME SERIES ANALYSIS

As pointed out in the previous section there is a natural connection between the stock prices' dispersion and the excess return from cointegration tracking. This section investigates their relationship in a time series

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context. Since the stock price dispersion is integrated of order one,[5] a basic stationary specification relates the abnormal return (AR) to the daily change in stock price dispersion (DD), including also the lagged abnormal return and some lagged changes in dispersion, together with the squared lagged change in dispersion, to account for potential asymmetries:

$$AR_t = \alpha + \beta_1 AR_{t-1} + \beta_2 DD_t + \beta_3 DD_{t-1} + \beta_4 DD_{t-2} + \beta_5 DD_{t-1}^2 + \varepsilon_t \quad (3)$$

The estimation results based on the DJIA sample from 4th January 1992 to 31st December 2001 are presented in Table 2. Significant coefficients are associated with the lag of the excess return, the first lag and the squared lagged change in dispersion. The positive coefficient of the lagged excess return accounts for the autocorrelation in the abnormal return. The contemporaneous and the second lag of the change in dispersion are not statistically significant, but there is a negative and very significant relationship between the abnormal return and the first lagged change in dispersion. Thus following an increase in dispersion the portfolio will make a loss relative to the market. The significant positive coefficient on the squared change in dispersion term indicates a non-linearity in this relationship: the larger the absolute change in dispersion, the higher the abnormal return.

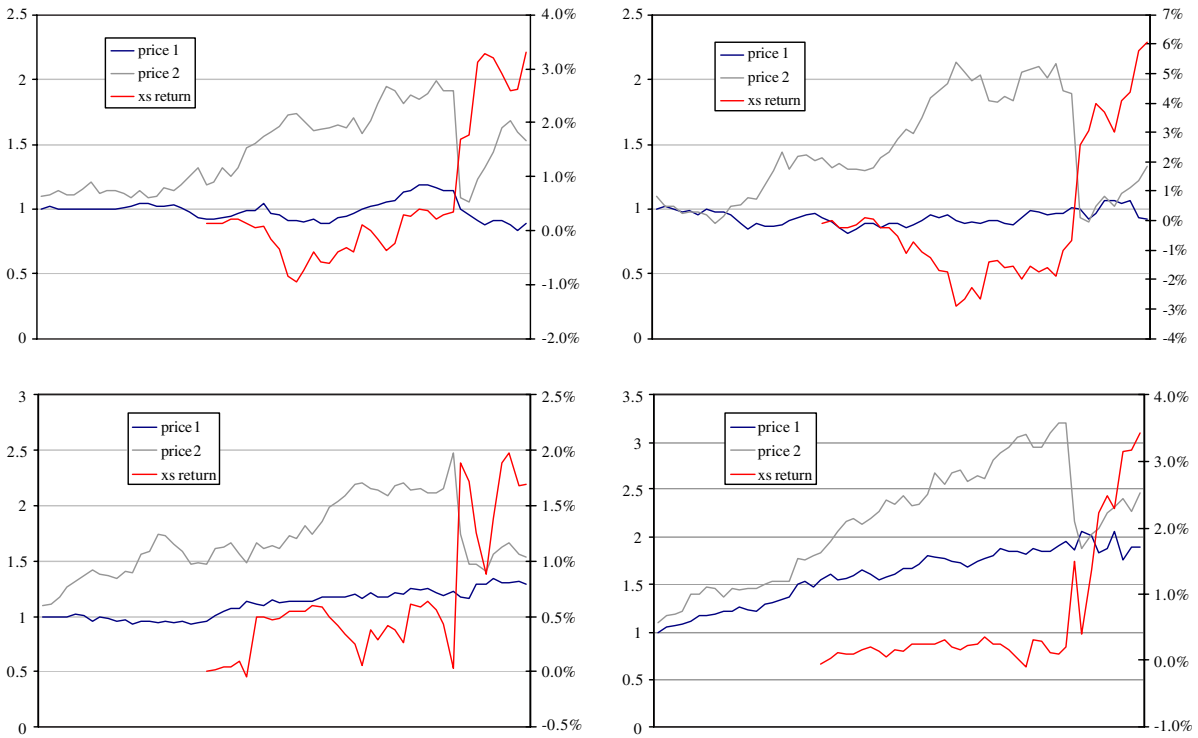


Figure 3. Abnormal return for some simulated rational speculative bubbles.

Table 2. Estimation of model (4)

	α	β_1	β_2	β_3	β_4	β_5
Coefficient	−0.00003	0.071907	0.001061	−0.01842	0.004874	0.406344
Standard error	0.000033	0.019675	0.002336	0.002346	0.002368	0.052512
t-statistic	−0.94859	3.654681	0.454261	−7.85479	2.058311	7.738151
p-value	0.3429	0.0003	0.6497	0	0.0397	0

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Table 3. Stability test for model (4)

Wald stability test: 2227 (October 16, 2000)				
F-statistic	358.01		Probability	0.000000
Jan-92 to Oct-00	α	β_1	β_3	β_5
Coefficient	0.000036	0.007495	−0.05627	0.038299
Standard error	0.000025	0.018067	0.001942	0.045035
t-statistic	1.405222	0.414814	−28.9779	0.85042
p-value	0.1601	0.6783	0	0.3952
Oct-00 to Dec-01	α	β_1	β_3	β_5
Coefficient	−0.00010	0.047449	0.102299	0.596204
Standard error	0.000138	0.041945	0.006819	0.139306
t-statistic	−0.74385	1.131212	15.00124	4.279809
p-value	0.4576	0.2589	0	0

Given the asymmetry in stock markets (i.e. stock prices tend to fall faster than they rise), a sudden large change in dispersion is likely to happen during market crises rather than during stable trending markets. Thus the highly significant positive value for β_5 confirms our initial hypothesis of a connection between the abnormal return and market crises periods. Finally, the fact that the excess return is determined by the *lagged* change in dispersion rather than by a simultaneous variable indicates that dispersion may be a useful leading indicator of the performance of this strategy.

However, on further investigation the structural stability of this relationship seems questionable. A rolling Wald test indicates that the null hypothesis of no structural break is most significantly rejected on 16th October 2000. Consequently, two separate regressions are estimated, using data before and after this point, and these have quite different results (Table 3). Note that, when the impact of the change in dispersion is separated into two subsamples, the lagged dependent variable becomes insignificant in both cases. Hence in Table 2 it was merely capturing the structural break that had been omitted from the model and it would be naïve to conclude from Table 2 that abnormal returns are the result of an autocorrelation induced by overlapping in-sample periods. The main difference between the two subsamples is the sign of the coefficients of the lagged dispersion. The slope coefficient of the lagged change in dispersion is, until 16th October 2000, very significant and negative, but after this the relationship between the two variables becomes even more significant, and *positive*. The coefficient of the squared lagged change in dispersion is not significant in the first period, but significantly positive in the second subsample.

5. STRATEGIC POSITIONING AND STOCK MARKET REGIMES

The parameter stability results have demonstrated that a very significant change in the behaviour of the abnormal return occurred in October 2000. Why then? To answer this we take a closer look at the US markets at this time. First, it was a period of great uncertainty, running up to the US presidential elections. Interest rates were expected to fall, having been rising prior to 2000 but stable since then. And perhaps most importantly, this period marked the time of the second great fall in the Nasdaq composite index. Index volatility reached 47.59% and the index fell 48.25% (i.e. 745.83 points), having already fallen 425 points from March 2000. Therefore, it is reasonable to infer that October 2000 marked the end of the technology bubble.

The situation depicted in Figure 4 resulted as, following the burst of the technology bubble, the technology stock prices fell below the equilibrium prices, which were based on the very high levels of dispersion observed previously. In Figure 4, without loss of generality, we have used the example of a higher than average priced stock. For reasons of space we leave to the reader the equivalent intuition based on a lower than average priced stock. The section entitled 'regime one' in the figure shows a stable (and upwards trending) market with a smooth line representing the long-run equilibrium price of this stock, on

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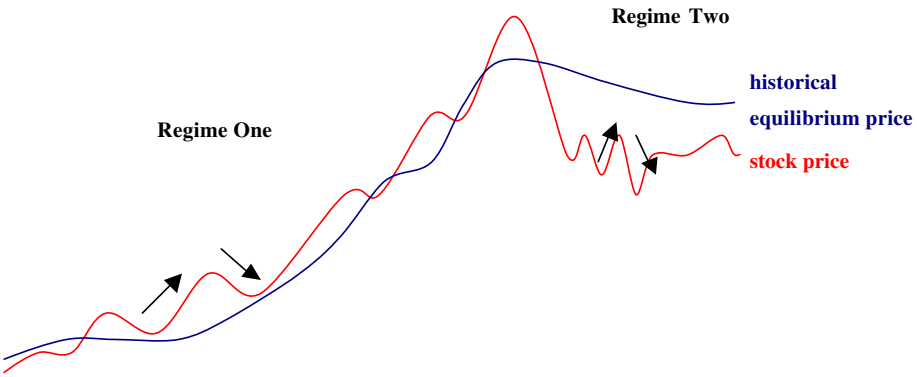


Figure 4. Stock price movements in regimes one and two.

which the cointegration weights are constructed, and a wavy line representing the actual price of this stock, determining its benchmark weight.[6]

Why should abnormal return have a negative relationship with lagged dispersion before October 2000, and a positive relationship thereafter? Again, Figure 4 offers some intuition behind this finding. If the actual price of the stock increases, diverging from the rest of the stock prices, its weight in the benchmark will also increase. However, its weight in the cointegration portfolio, being based on a long history of prices, will not react immediately to the increase in the stock price, which could be just noise from a long-run equilibrium perspective. This also explains the negative relationship with the momentum factor identified at a monthly frequency. The portfolio will be relatively underweighted on this particular stock while its price is increasing, realizing relative losses compared to the benchmark. And because the stock has higher than average price, whilst these relative losses are made, the dispersion is increasing. When the price increase reverts and the stock price starts falling, returning towards its long-run equilibrium level, the dispersion in the system also decreases whilst the portfolio will make a relative profit compared with the benchmark, because it is still underweighted (relative to the benchmark) on a stock whose price is declining. Thus, dispersion should have a *negative* relationship with excess return in a stable market.

Now consider the section titled 'regime two', which illustrates the stock price after an upward trend in the stock price followed by a sharp price decrease. The effect of the price fall is that (a) the cointegration relationship is now based on a historical equilibrium with a price well above the actual stock price shown by the wavy line, and (b) the dispersion in the index has decreased (because before the price fell, the stock had an above average price). The cointegrating portfolio will be overweighted in this stock, relative to the benchmark, and the portfolio will realize a relative profit if the stock price increases. So in the 'post-crash' regime dispersion will have a *positive* relationship with excess return. Thus if dispersion increases during the recovery period, the cointegration strategy will generate abnormal returns.

Note that significant abnormal returns relative to the benchmark can be made *during* a period of sharp trend reversals. If prices rise in a stable market, this gives time for the cointegration based portfolio to incorporate the new price information in portfolio weights; they move towards the weights in a price weighted benchmark and the cointegrating portfolio tracks the benchmark closely during such times. But price falls can be sudden, and they often precipitate further price falls. A sharp reversal in trend can occur after the prices of some stocks (e.g. technology stocks) have risen more rapidly than others. At the same time as the dispersion increases, the cointegrating portfolio will be based on an outdated equilibrium where these stocks are underweighted relative to the benchmark. The portfolio begins to adopt a market timing bet, so that *if* prices subsequently fall with prices of overvalued stocks falling more than prices of other stocks, the portfolio will realize abnormal returns as the dispersion decreases. During stable markets with no indication of a regime change in the future, the portfolio will track the benchmark effectively. However if there are signs of the formation of a speculative bubble the portfolio either continues to track the benchmark or—especially if the bubble grows very quickly—accepts a small relative loss. At such times

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there is an implicit element of strategic positioning in the portfolio, as it 'bets' on a regime change that will bring abnormal returns to its investors.

In our empirical example, before the technology crash, the prices of technology stocks were far too high relative to the prices of traditional stocks. This led to a high dispersion in the early part of 2000, which persisted in the structure of the cointegrating portfolio even after the dispersion of the actual prices had decreased, following the burst of the technology bubble. Thus during the crash and, since the dispersion subsequently increased again, during the upturn period following the crash, the cointegration portfolio made significant excess returns.

Of course, the symmetric situation, where unintentional abnormal losses would occur following a market 'hike', is *theoretically* possible. However, it is a well-documented stylized fact of equity markets that stock prices usually fall more rapidly than they rise. This is the result of the leverage effect, frequently documented in equity markets (Black, 1976; Christie, 1982; French *et al.*, 1987) and the presence of positive feedback: an initial sell reaction to some bad news will be followed by more selling, driving the prices faster below their fundamental levels (De Long *et al.*, 1990). In the presence of such asymmetry, it comes as no surprise that mostly *positive* abnormal returns occurred in all the real and simulated markets that we have considered.

6. A MARKOV SWITCHING MODEL

There is a clear time-variability in the parameters of the estimated regressions in Table 3, which can be accounted for with simple tools like structural break tests, but only at the expense of inducing a significant degree of arbitrariness. There appear to be some grounds for a structural break in the relationship between the abnormal return and dispersion in October 2000, but this does not ensure that the break identified is unique. To address these issues, we employ a Markov switching framework.

Belonging to a very general class of time series models, which encompasses both non-linear and time-varying parameter models, the regime switching models provide a systematic approach to modelling multiple breaks and regime shifts in the data generating process. Increasingly, regime shifts are considered to be governed by exogenous stochastic processes, rather than being singular, deterministic events. When a time series is subject to regime shifts, the parameters of the statistical model will be time varying, but in a regime switching model the process will be time-invariant conditional on a state variable that indicates the regime prevailing at the time.

The importance of these models has long been accepted, and the pioneering work of Hamilton (1989) has given rise to a huge research literature (Hansen, 1992, 1996; Kim, 1994; Diebold *et al.*, 1994; Garcia, 1998; Psaradakis and Sola, 1998; Clarida *et al.*, 2003). Hamilton (1989) provided the first formal statistical representation of the idea that economic recessions and expansions influence the behaviour of economic variables. He demonstrated that real output growth might follow one of two different autoregressions, depending on whether the economy is expanding or contracting, with the shift between the two states generated by the outcome of an unobserved Markov chain.

In finance, the applications of Markov switching techniques have been many and very diverse, including modelling state-dependent returns (Perez-Quiros and Timmermann, 2000) and volatility regimes (Hamilton and Lin, 1996), term structure models of interest rates (Clarida *et al.*, 2003, 2004), exchange rate intervention (Taylor, 2003), monetary policy (Melvin *et al.*, 2004), option pricing (Aingworth *et al.*, 2002), detecting financial crises (Coe, 2002), bull and bear markets (Maheu and McCurdy, 2000) and periodically collapsing bubbles (Hall *et al.*, 1999), and measuring mutual fund performance (Kosowski, 2001). Despite their limited forecasting abilities (Dacco and Satchell, 1999), Markov switching models have been successfully applied to constructing trading rules in equity markets (Hwang and Satchell, 1999), equity and bond markets (Brooks and Persand, 2001) and foreign exchange markets (Duecker and Neely, 2001).

The Markov switching model specified for the excess return assumes the presence of a latent variable (state variable), which determines the form of linear relationship between the abnormal return and the lagged dispersion in stock prices. The advantages of using a latent variable approach instead of a pre-defined indicator have been long documented. For example, when analysing business cycles, the Markov

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switching model produces estimates of the state conditional probabilities, which contain more precise information about the states that are driving the process than a simple binary indicator of the states, which is prone to significant measurement errors. The estimates of the conditional probability of each state allow more flexibility in modelling the switching process. An additional motivation for using a latent variable approach in this case is the fact that there is no obvious indicator of the states of the process generating the abnormal return—the price dispersion appears to be the leading indicator of the abnormal return, but we have no prior on the determinant of the regime switches.

In the Markov switching model of abnormal return, the intercept, regression slope and the variance of the error terms are all assumed to be state-dependent. If we let S_t denote the latent state variable, which can take one of two possible values (i.e. 1 or 2), then the model can be written as:

$$\mathbf{y}_t = \mathbf{z}_t' \boldsymbol{\beta}_{S,t} + \boldsymbol{\varepsilon}_{S,t} \tag{4}$$

where

$\mathbf{y}_t$ = vector of the excess returns
$\mathbf{z}_t = (\mathbf{1}, \mathbf{x}_t, \mathbf{x}_t^2)$, the matrix of explanatory variables
$\mathbf{x}_t$ = vector of lagged change in the prices dispersion[7]
$\boldsymbol{\beta}_{S,t} = (\boldsymbol{\mu}_{S,t}, \boldsymbol{\gamma}_{S,t}, \boldsymbol{\theta}_{S,t})$ is the vector of state-dependent regression coefficients
$\boldsymbol{\varepsilon}_{S,t}$ = vector of disturbances, assumed normal with state-dependent variance $\boldsymbol{\sigma}_{S,t^2}$

The transition probabilities for the two states are assumed to follow a first-order Markov chain and to be constant over time:

$$P\{S_t = j | S_{t-1} = i, S_{t-2} = l, \ldots\} = P\{S_t = j | S_{t-1} = i\} = p_{ij}$$

The matrix of transition probabilities can be written as:

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix} = (p_{ij})$$

If we let $\boldsymbol{\xi}_t$ represent a Markov chain, with $\boldsymbol{\xi}_t = (1, 0)'$ when $S_t = 1$ and $\boldsymbol{\xi}_t = (0, 1)'$ when $S_t = 2$, then the conditional expectation of $\boldsymbol{\xi}_{t+1}$ given $S_t = i$ is given by:

$$E(\xi_{t+1} | S_t = i) = \begin{pmatrix} p_{i1} \\ p_{i2} \end{pmatrix} = \mathbf{P}\xi_t$$

The conditional densities of $\mathbf{y}_t$, assumed to be Gaussian, are collected in a 2×1 vector: $\boldsymbol{\eta}_t = (\eta_{1t}, \eta_{2t})$ where $\eta_{1t} = f(\mathbf{y}_t | S_t = i, \mathbf{z}_t; \boldsymbol{\alpha})$ is the normal density function whose parameters $\boldsymbol{\alpha}$ are conditional on the state. That is, $\eta_{it} = [(2\pi)^{1/2}\sigma_i]^{-1}\exp(-(\mathbf{y}_t - \mathbf{z}_t'\boldsymbol{\beta}_i)^2/2\sigma_i^2)$. The conditional state probabilities can be obtained recursively:

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \otimes \eta_t}{\mathbf{1}'(\hat{\xi}_{t|t-1} \otimes \eta_t)} \tag{5}$$

$$\hat{\xi}_{t+1|t} = \mathbf{P}\hat{\eta}_{t|t}$$

where $\hat{\xi}_{t|t}$ represents the vector of conditional probabilities for each state estimated at time t, based on all the information available at time t, while $\hat{\xi}_{t+1|t}$ represents the forecast of the same conditional probabilities based on the information available at time t for time $t + 1$. The symbol $\otimes$ denotes element-by-element multiplication. The ith element of the product $\hat{\xi}_{t|t-1} \otimes \eta_t$ can be interpreted as the conditional joint distribution of $\mathbf{y}_t$ and $S_t = i$. The numerator in expression (5) represents the density of the observed vector $\mathbf{y}_t$ conditional on past observations. Given the assumptions made on the conditional density of the disturbances, the log likelihood function can be written as:

$$L(\boldsymbol{\alpha}, \mathbf{P}) = \sum_{t=1}^{T} \log f(\mathbf{y}_t | \mathbf{z}_t; \alpha; \mathbf{P}) = \sum_{t=1}^{T} \log \mathbf{1}'(\hat{\xi}_{t|t-1} \otimes \eta_t)$$

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This approach allows the estimation of two sets of coefficients for the regression and variance of the residual terms, together with a set of transition probabilities.

Considering the complexity of the log likelihood function and the relatively high number of parameters to be estimated, the selection of starting values is critical for the convergence of the likelihood estimation. To reduce the risk of data mining, we have not used any state-dependent priors as starting values. Instead, we have used the unconditional estimates of the regression coefficients and the standard error of the residual term. Additionally, we have arbitrarily set $\xi_{1|1}$ to $(1,0)'$. A number of restrictions need to be imposed on the coefficient values, in order to ensure their consistency with model assumptions. The transition probabilities were restricted to be between 0 and 1, while a non-negativity constraint was imposed on the standard deviation of the residuals in both states.

The data covered 10 years of daily observations from 1992 to 2001. Table 4 reports the Markov switching model estimation results over the entire sample. The only coefficient that is not statistically significant at 1% is the regression intercept, for the second state. A noteworthy difference between the two states concerns the coefficients of the lagged and squared lagged change in dispersion: in the second state the coefficients are positive while in the first state they are negative, thus the lagged change in dispersion has a different effect on the abnormal return in the two states. In the second state an increase in the index dispersion is followed by a relative gain from index tracking in the next period, and the larger the change the higher the abnormal return, while in the first state a decrease in dispersion is associated with relative gains. The negative coefficient of the squared lagged change in dispersion in the first state can be the effect of a predominance of dispersion changes in the direction of the trend, which should be negatively related to the abnormal return. In this context, the negative sign of the squared lagged change in dispersion is highly sample-dependent.

Additionally, the standard deviation of the residuals is higher in the second state. Apart from higher volatility (6.4% p.a., as compared to 1.8% in regime one) and higher returns, state two returns have a positive skewness (0.99) and higher excess kurtosis (3.42). State one returns are closer to normality (skewness 0.09; excess kurtosis 2.18). Regarding the transition probabilities, both states are very persistent (close to those of a reducible Markov switching model) even if the first state appears to be slightly more persistent than the second one. The probability of staying in state one at time $t+1$, given that at time t the process was in state one, is 0.998, while the probability of remaining in state two, once there, is 0.996. Standard likelihood ratio tests clearly reject the null hypothesis that either p_{11} or p_{22} are equal to one ($\mathrm{LR}_1 = 128.5$ and $\mathrm{LR}_2 = 130.99$).

If we split the sample observations between the two states using the criterion of estimated probability,[8] we can determine the abnormal return associated with each state. Based on this procedure the number of observations in state one is almost three times the number of observations in state two. Figure 5 shows the cumulative excess return from each state: there is a consistent abnormal return produced in state two. However, the first state produces small positive returns followed by small negative returns over cycles of six to eighteen months. Based on the same separation procedure as above, we observe that the benchmark generated smaller returns in state two than in state one (the equivalent of 4.90% p.a. as opposed to 14.36% p.a.). Another notable difference concerns the volatility of the benchmark during each state: state two returns are associated with an annual volatility of 19.8%, while the benchmark returns corresponding to state one have only 13.4% annual volatility. Therefore the abnormal returns occur in periods with lower returns and higher volatility for the market.

Table 4. Estimation of model (5)

	μ_1	μ_2	γ_1	γ_2	θ_1	θ_2	σ_1	σ_2	P_{11}	P_{22}
Coefficient	3.47E-05	−1.1E-04	−0.0571	0.0439	0.0343	0.8755	0.00095	0.00275	0.9987	0.9963
Standard error	0.0000	0.0001	0.0013	0.0045	0.0460	0.0681	0.0000	0.0000	0.0634	0.0729
Z-statistic	1.4621	−0.8025	−42.642	9.8054	0.7454	12.849	529.210	248.715	15.740	13.673
p-value	0.1437	0.4223	0	0	0.456	0	0	0	0	0

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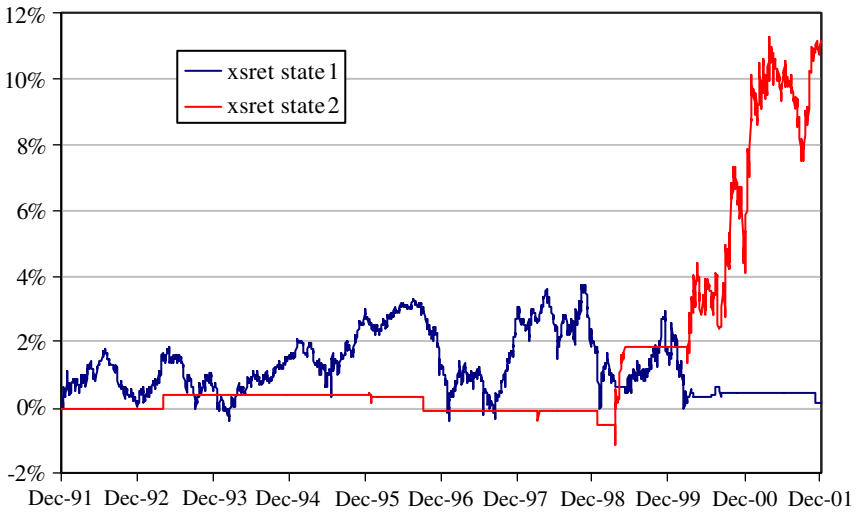


Figure 5. Regime conditional abnormal returns.

Our conclusion is that the two states have very distinctive characteristics: state two, which occurs less frequently but is predominant during the last few years, is responsible for producing the entire abnormal return. This state occurs in more volatile market conditions and the over-performance is subsequent to an increase in the index dispersion. In the first state there is a negative, but not significant, excess return with any positive excess return occurring further to a decrease of the index dispersion.

7. WHAT DRIVES THE REGIMES?

Why does the equity market change from the 'normal' regime with no abnormal returns to the high volatility regime with abnormal returns, and back again? Is it possible that the regime switch could be driven by macroeconomic factors, such as interest rates or growth rates, or market microstructure variables such as information flows? In simple autoregressive Markov switching models this type of question can be addressed by introducing endogenous transition probabilities (see for instance Taylor, 2003; Melvin *et al.*, 2004). However, this is less straightforward in a multivariate Markov switching model and instead we follow the approach of Clarida *et al.* (2004), where our estimated state probability from model (4) is subsequently related to explanatory variables using a binary choice model.

Figure 6 shows how the probability of state two evolves over time.[9] It is clear that state one is prevailing at the beginning of the sample, but state two becomes predominant towards the end of the sample. Over the entire data sample, observations in state two represent only 18% of the total number of observations but in 2000 and 2001, state two occurs 84% of the time. Figure 6 shows that marked increases in the probability of the high volatility market regime occurred in: September 2001, which was the market crisis precipitated by the 9/11 terrorist attack; October 2000, which coincided with the lead up to the US presidential election in November 2000. The equity market had stabilized following the technology crash and a change in the policy of high interest rates was anticipated following the election; April 2000 marked the end of the technology boom. After a long period of rising interest rates, these started to level off. Also, the credit spread rose significantly (e.g. the Goldman Sachs 10-year BBB spread increased from 60 bps to 100 bps). Finally, in April 1999 a clear upward trend in credit spreads can also be seen as investors feared an increase in company defaults with more and more new companies issuing bonds to raise capital on the basis of overvalued share prices.

A positive relationship between credit spreads and the probability of regime two is apparent throughout the latter part of the sample. In the inset box in Figure 6 we see that the regime two probability increases

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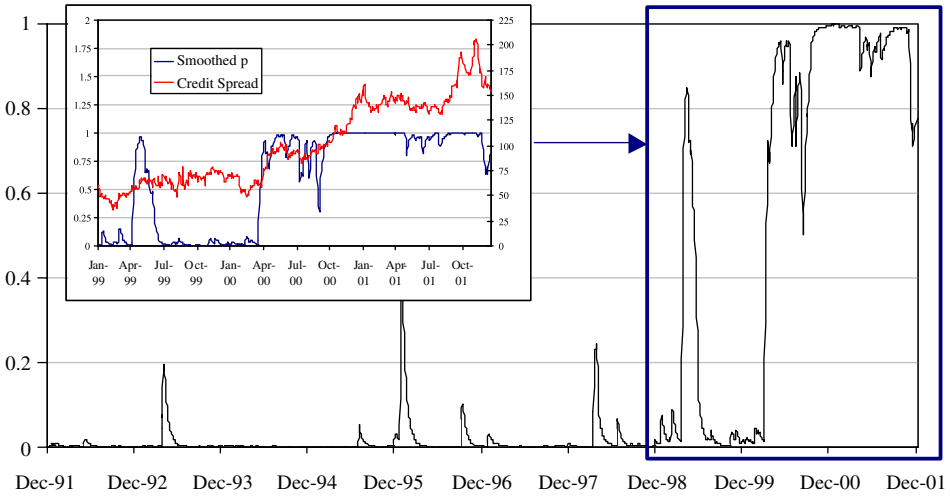


Figure 6. Estimated probability of regime two (exponentially smoothed with $\lambda = 0.90$).

Table 5. Estimation of model (6)

	δ_0	δ_1
Coefficient	−7.3271	0.0919
Standard error	0.5669	0.0073
Z-statistic	−12.9235	12.5543
p-value	0.0000	0.0000

with the 10-year BBB credit. A simple logit model is used to quantify this relationship:

$$p_t = \frac{\exp(d_0 + d_1 c_t)}{1 + \exp(d_0 + d_1 c_t)} \tag{6}$$

where c_t is the credit spread and p_t is the (smoothed) probability of regime two at time t.

Table 5 reports the estimation results for the sample period January 1999 to December 2001. Both the intercept term and the coefficient of the spread are statistically significant at 5%, the latter indicating that there is a clear positive relationship between regime two probability and the credit spread, with an R-squared of 0.56.

We have considered whether other macroeconomic and microeconomic variables are related to the regime probability. For instance, the high volatility state could result from an increase in information flows, with abnormal returns arising as agents become more informed. However, only the credit spread was found to have any empirical significance. Low grade credit spreads increase as economic growth slows and equity prices fall,[10] so the credit spread is a useful indicator of the equity market regime because the features of the return on equities are so different in the two regimes.

8. STATISTICAL INFERENCE AND MODEL PREDICTIVE POWER

Whilst there is clear evidence that the abnormal return has different behaviour in the two states, this does not imply that the asymmetries between the two states are also statistically significant. To validate our

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inferences on the process driving the abnormal return one needs to test and reject the null hypothesis of no switching. Standard testing methods such as likelihood ratio tests are not applicable to Markov switching models due to the presence of nuisance parameters under the null hypothesis of linearity, or no switching. The presence of nuisance parameters gives the likelihood surface sufficient freedom so that one cannot reject the null hypothesis of no switching, despite the fact that the parameters are apparently significant.

A formal test of the Markov switching models against the linear alternative of no switching, which is designed to produce valid inference, has been proposed by Hansen (1992, 1996). Following Hamilton and Lin (1996), we have generated Hansen's statistic for M values of 0 to 4 and found that the null hypothesis is strongly rejected with a p-value of 0.0000. An alternative approach to the Hansen statistic uses a classical log likelihood ratio test for estimating (a) the asymmetries in the conditional mean, assuming the existence of two states in the conditional volatility, and (b) the asymmetries in the conditional volatility, assuming the existence of two states in the conditional mean. Such a test follows the standard chi-squared distribution. We have tested the following hypotheses: (1) the intercept and slope coefficients are not significantly different between the two states [H_0: $\mu_1 = \mu_2$; $\gamma_1 = \gamma_2$; $\theta_1 = \theta_2$] and (2) the standard deviations of the residuals of the two states are not significantly different [H_0: $\sigma_1 = \sigma_2$]. Both tests indicated a rejection of the null hypothesis at the highest significance, with LR statistics of 635.0 and 935.6, respectively. Therefore we conclude that there is clear evidence of asymmetries between the two regimes identified by the model and these are not only economically but also statistically significant.

It is well known that standard out-of-sample testing methods are not applicable to Markov switching models due to the presence of nuisance parameters (Hansen, 1992). Thus in order to test the out-of-sample predictive power of the model we follow standard practice to use an operational criteria (in our case, the construction of a trading rule) instead of a statistical criteria. To this end we extend our initial database up to November 2002 and propose two strategies that are designed to exploit the regime-dependent relationship between the index tracking over-performance and the stock price dispersion. Their construction is based on the fact that the *lag* of the change in dispersion is used to explain the abnormal return and, therefore, we have a leading indicator of portfolio performance. Also, forecasts of the latent state conditional probability can be produced for a number of steps ahead by using the unconditional transition probabilities and the current estimate of the conditional probabilities of the latent states.

The portfolio P generating the abnormal return relative to the index is defined as the difference between the replica portfolio holdings and the benchmark holdings in each stock. Both trading rules assume active trading, with daily rebalancing according to a trading signal. The first trading rule ensures that P is held only if there is a buy/hold signal from the Markov switching model. In the second trading rule, P is held if there is a buy/hold signal, and is shorted otherwise. The 'buy/hold' signal occurs either after an increase in the dispersion, if the forecast of the conditional probability of the latent state indicates that the process is currently in regime one, or after a decrease in dispersion, if the forecast of the conditional probability indicates that the process is currently in regime two. Note that the trading rule is constructed solely on the sign of the change in dispersion and not on its magnitude, although potentially this could be used for rebalancing filters. As the abnormal return is not correlated with the market return both strategies will inherit market neutral characteristics. Moreover, they are self-financed, as the sum of all stock weights in P is zero by construction. In order to obtain the signal for a given date we have used only the information available at the moment of the signal estimation (the sign of the lagged change in dispersion and the one-period ahead forecast of the conditional probability of the regimes).

The returns on these trading strategies, without accounting for transactions costs, are plotted in Figure 7. We have not accounted for potential transaction costs here, as we only aim to test the predictive power of the model. Over the 11-month testing period the first trading rule produced a cumulative return of 5.9% with an average annual volatility of 2.3% and the second trading rule produced over the same time interval a cumulative return of 13.4% with a slightly higher average annual volatility, 3.1% p.a. The cumulative returns to both trading strategies demonstrate the predictive power of the Markov switching model. However from a trading perspective these results should be treated with caution. The very high profitability of the trading rules could be the result of the predominance and persistence of regime two during this period. Moreover the rules require daily rebalancing and this could result in significant transaction costs

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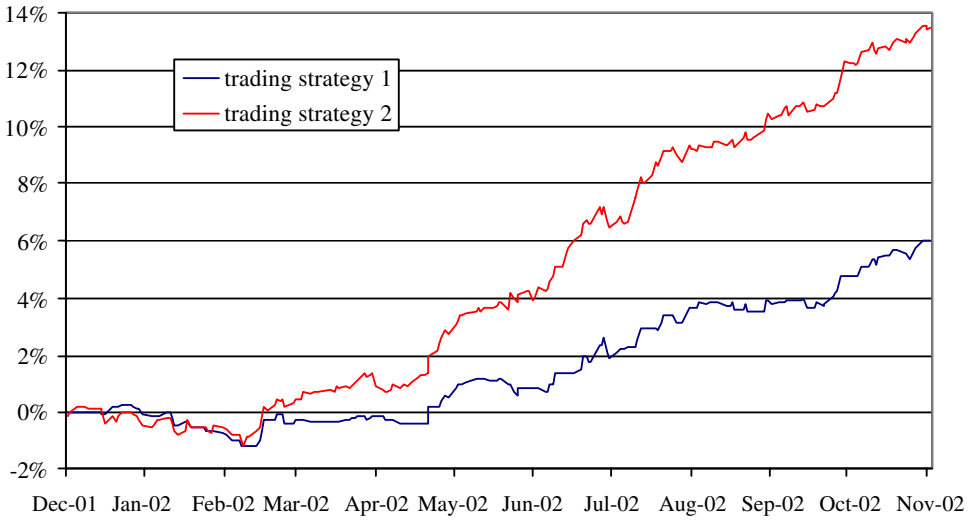


Figure 7. Cumulative returns produced by the trading rules.

that erode any apparent profitability. The problem of potentially high transaction costs in trading rules based on Markov switching forecasts is not new and has been dealt with either by reducing the frequency of trades, or by imposing some filtering of the signals, when trades take place only if the signal exceeds a given threshold (Duecker and Neely, 2001).

In summary, this section has established the appropriateness of the Markov switching approach for modelling the abnormal return. Although the tracking portfolio was shown to replicate the index accurately most of the time and outperform it in given market circumstances—so the cointegration model is a true enhanced tracking strategy—attempts to time the abnormal return through a dynamic market neutral strategy based on a Markov switching model may not justify the costs involved.

9. IMPLICATIONS FOR MARKET EFFICIENCY

This section discusses the implications of our findings for market efficiency. We have provided evidence of consistent outperformance from the cointegration based tracking strategy, even after transaction costs, in different stock universes and over different time periods. The cointegration portfolio was shown to replicate its benchmark accurately most of the time and to over perform it in special, more volatile, market circumstances. Having identified the mechanism producing the abnormal return we conclude that the outperformance is sample specific only to the extent that it is associated with specific market circumstances, indicating a transitional period for prices. Moreover, we have found a leading indicator for the abnormal return within a Markov switching framework.

However, the success of the cointegration strategy to exploit the information in past stock prices and the predictive power of the Markov switching model can only be interpreted as evidence against the efficient market hypothesis in the weak form if, and only if, the abnormal return does not represent a hidden risk factor premium.[11] As shown by Cochrane (1999) in a world in which there are multiple sources of priced risk, the multifactor efficient market portfolio will no longer be on the mean–variance efficient frontier and will appear to be dominated to an investor interested only in mean–variance. In this context, if the Markov switching model is detecting a hidden risk factor then market inefficiency has not been proved. On the other hand, if such a hidden risk factor does not exist, then the Markov switching model is identifying pricing

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inefficiencies that are exploited by the cointegration strategy, albeit only temporary and occurring when the market returns are low and the volatility is high.

We have shown that in special, transitional market circumstances the cointegration tracking portfolio develops a strategic position corresponding to a bet that the prices of stocks that have deviated far above their equilibrium level will revert to their historical averages. Thus the 'abnormal' return is in fact the result of a successful market timing bet, by definition not risk free. One these grounds one cannot claim market inefficiency.

10. SUMMARY AND CONCLUSIONS

This paper has demonstrated that excess returns can be generated through a very simple dynamic indexing strategy and we have found a leading indicator for the abnormal return, a measure of stock price dispersion. The non-linear relationship between the abnormal return and lagged dispersion required a Markov switching approach, and this has revealed the existence of two stock market regimes having very distinctive characteristics. We found that almost the entire abnormal return was associated with the regime characterized by higher benchmark volatility and lower benchmark returns and that this regime is likely to occur when credit spreads are particularly high. Hence investors that seek to diversify the risk of passive indexation when equity returns are low but wish to avoid the credit risk of high yield bonds, should find a suitable alternative investment in this type of enhanced indexation.

The enhanced indexation is based on a cointegration relationship between the portfolio and a benchmark. It can be interpreted as a relative pricing model with an implicit market timing element that pays off if the market switches from the regime with stable returns and low volatility to the high volatility regime when the benchmark returns are low. During the stable regime, for as long as stock prices are oscillating around the past equilibrium levels, the strategy generates accurate replicas of the benchmark. But in volatile markets, when returns are low and prices are moving towards new levels, the strategy produces consistent excess returns. We have argued that the reason why the cointegration strategy has some periods when it significantly over-performs its benchmark but no periods when it significantly under performs it, is the asymmetric behaviour of stock prices, the fact that prices tend to fall faster than they rise. The result is that the cointegration portfolio successfully exploits general stock market declines and recovery periods even though it is not specifically designed for this purpose.

We have shown that the abnormal return occurs only during especially volatile periods, so we cannot exclude the fact that the abnormal return represents a risk premium. Thus we find no evidence against market efficiency. Even if such a risk factor does not exist, the anomalies identified by cointegration have been shown to be only temporary and to occur only in special market circumstances. Nevertheless, our findings do have wide implications for equity fund managers. We have shown that, without any stock selection, solely through smart optimization that has an implicit element of market timing, the benchmark performance can be significantly enhanced in certain market circumstances. Moreover, the strategy can be implemented to replicate any type of value or capitalization weighted benchmark, not only wide market indexes.

NOTES

1. For example, in our case the benchmark is reconstructed as a price weighted index of the stocks currently in the DJIA, and so the reconstructed benchmark represents a scale adjustment to the DJIA based on the value of the latest index divisor.
2. Stock prices and market indexes are usually found to be integrated of order one. The preliminary analysis of our data, using standard augmented Dicky–Fuller tests, showed that this is also our case, i.e. all stock prices and the reconstructed index are I(1). Results are available from the authors on request.
3. The issue of the transaction costs is an important one, especially in connection with the EMH. We assume an amount of 20 basis points on each trade value to cover the bid–ask spread and the brokerage commissions, which is conservative for very liquid stocks such as the ones in DJIA. The transaction costs estimated over the entire data sample sum up to no more than 2.5%. Such an amount of transaction costs can hardly be regarded as affecting the overall performance of the strategy.

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4. The total number of simulated data points is 60, with the bubble collapsing at observation 51. Note that, in the case of cointegration based portfolios, given that the sample used to calibrate the portfolio is much larger, the historical prices effect should be even more pronounced than it is in the simulations.
5. The ADF statistic for the dispersion series is -0.81, thus the null hypothesis of unit root cannot be rejected (5% critical value is -2.86). The ADF statistic for the first difference in dispersion, however, is -22.10, clearly rejecting the unit root hypothesis.
6. Figure 5 illustrates the stock price to be stable upwards trending because 'regime two' concerns a price fall after such an upwards trend. But in fact, 'regime one' can be any stable market, regardless of trend, as is evident from the reasoning in this paragraph.
7. The specification including lagged excess return as an explanatory variable, as expected from our simple regression results of Table 3, is not favourable according to information criteria, and the coefficients of the lagged excess return are not statistically significant.
8. If the estimated conditional probability of regime one at time t is above 0.5, we say that the process was in regime one at time t. Alternatively, the process will be in regime two.
9. The smoothed series is $p_t^* = (1-\lambda)p_t + \lambda p_{t-1}^*$ with $\lambda = 0.8$.
10. See Longstaff and Schwartz (1995), Duffee (1998), Collin-Dufresne *et al.* (2001) and many others.
11. The weak form of market efficiency assumes that at all times prices fully reflect the information comprised in past prices, as opposed to the other two forms of market efficiency, semi-strong and strong, which assume that prices also reflect all other public information, respectively, all other public and private information.

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